OBJECTIVE MATHEMATICS

Volume 2

Descriptive Test Series

Prof. M. L. Khanna
Bhushan Muley

CHAPTER-8 : DEFINITE INTEGRAL

UNIT TEST-1

1. Let (f) denote the greatest integer function. If $\int_0^{2.4}\left[x^2\right]dx = \alpha+\beta\sqrt{2}+\gamma\sqrt{3}+\delta\sqrt{5}$, then $\alpha+\beta+\gamma+\delta$ is equal to _____.

2. Let (f) denote the greatest integer $\le t$. Then $\frac{2}{x}\int_{\pi/6}^{5\pi/6}(8[\operatorname{cosec} x]-5[\cot x])dx$ is equal to _____.

Hints and Solutions

1. (6) $\int_0^{2.4}\left[x^2\right]dx\int_0^1 0\,dx+\int_1^{\sqrt{2}}dx+\int_{\sqrt{2}}^{\sqrt{3}}2dx$

$+\int_{\sqrt{3}}^{2}3dx+\int_2^{\sqrt{5}}4dx+\int_{\sqrt{5}}^{2.4}5dx$

$=0+(\sqrt{2}-1)+2(\sqrt{3}-\sqrt{2})+3(2-\sqrt{3})+4(\sqrt{5}-2)$

$+5(2.4-\sqrt{5})$

$=9-\sqrt{2}-\sqrt{3}-\sqrt{5}$

$\alpha=9,\beta=-1,\gamma=-1,\delta=-1$

2. (14)

$I=\int_{\pi/6}^{5/6}(8\cdot[\operatorname{cosec} x]-5[\cot x])\,dx$

$=8.\int_{\pi 6}^{5\pi/6}[\operatorname{cosec} x]dx-5\cdot\int_{\pi/6}^{5\pi/6}[\cot x]dx-8\,/_1-5\,/_2$

$I_1=\int_{\pi/6}^{5\pi/6}[\operatorname{cosec} x]dx=\int_{\pi/6}^{5\pi/6}1\cdot dx=\frac{2\pi}{3}$

$I_2=\int_{\pi/6}^{5\pi/6}[\cot x]dx=\int_{\pi/6}^{\pi/4}1\cdot dx+\int_{\pi/4}^{\pi/2}0\cdot dx$

$+\int_{\pi/2}^{3\pi/4}(-1)dx+\int_{3\pi/4}^{5\pi/6}(-2)dx$

$=-\frac{\pi}{3}$

Required value $=\frac{2}{\pi}l=\frac{2}{\pi}\left[8\cdot\frac{2\pi}{3}+5\cdot\frac{\pi}{3}\right]=14$